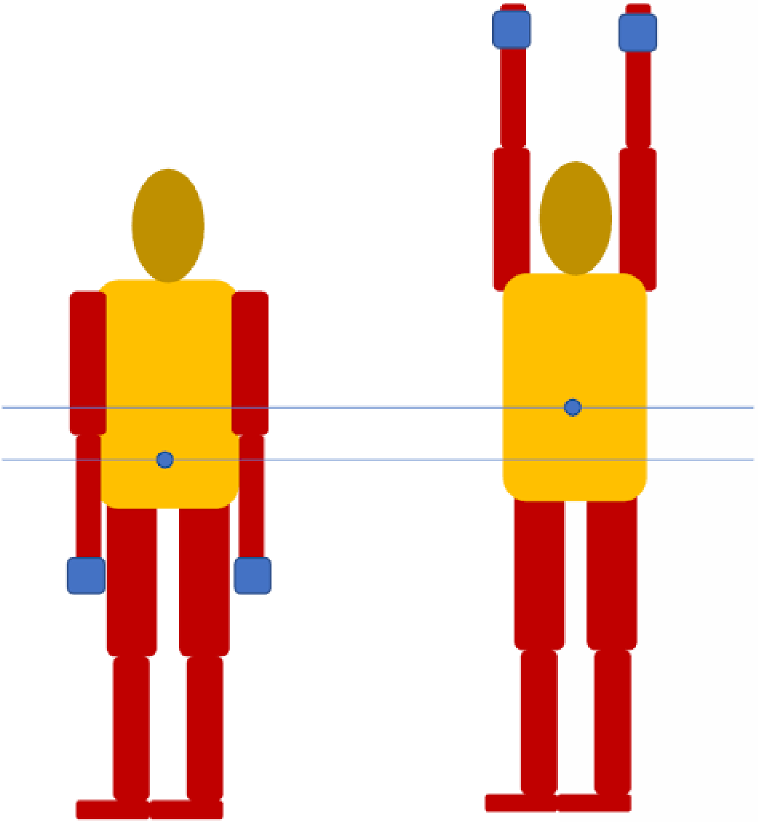


Grade 11S – Physics

Unit Two: Mechanics

Chapter 9: System of Particles

Prepared & Presented by: Mr. Mohamad Seif

OBJECTIVES

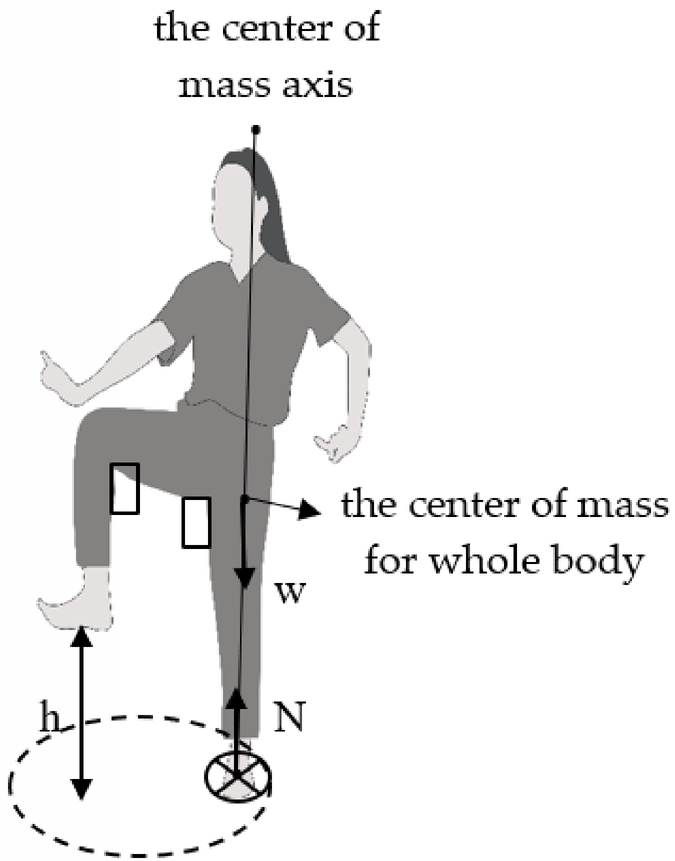
1. Define a system of particles.
2. Define Center of mass
3. Position vector of center of mass

Introduction

To fully understand mechanics of an object that consists of many moving parts, you will need to study motion of each part.

If you wanted to understand motion of a person, you will need to follow the motion of the arms, legs, head, etc…

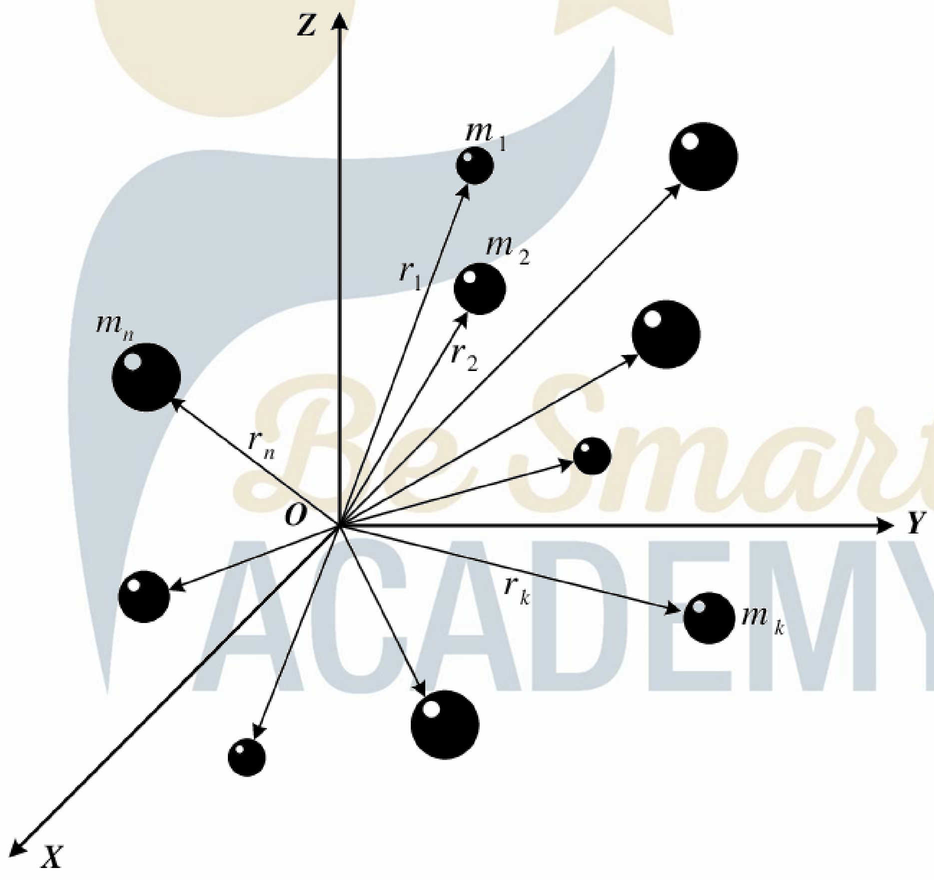
We can study overall translational motion of the entire body by replacing it by a single point and placing the total mass at that point.



This special point is called the center of mass

System of particles

A system of particles is a group of particles. Its size is considerable with respect to the distances it moves.



System of particles

System of Particles

Rigid body

Deformable body

System of particles

What is a rigid system?

A rigid system is the one in which the relative distance between the different particles forming the system remains constant during the motion of the system
In other words, a rigid system does not change its shape or size when subjected to external forces.

Concrete block

Steel bar

System of particles

What is deformable system?

A deformable system is the one in which the distance between its particles forming it can be changed during the motion of the system.

In other words, a deformable system change its shape or size when subjected to external forces.

Water Balloon

Rubber Band

Center of mass «CM»

What is the Center of mass (CM)?

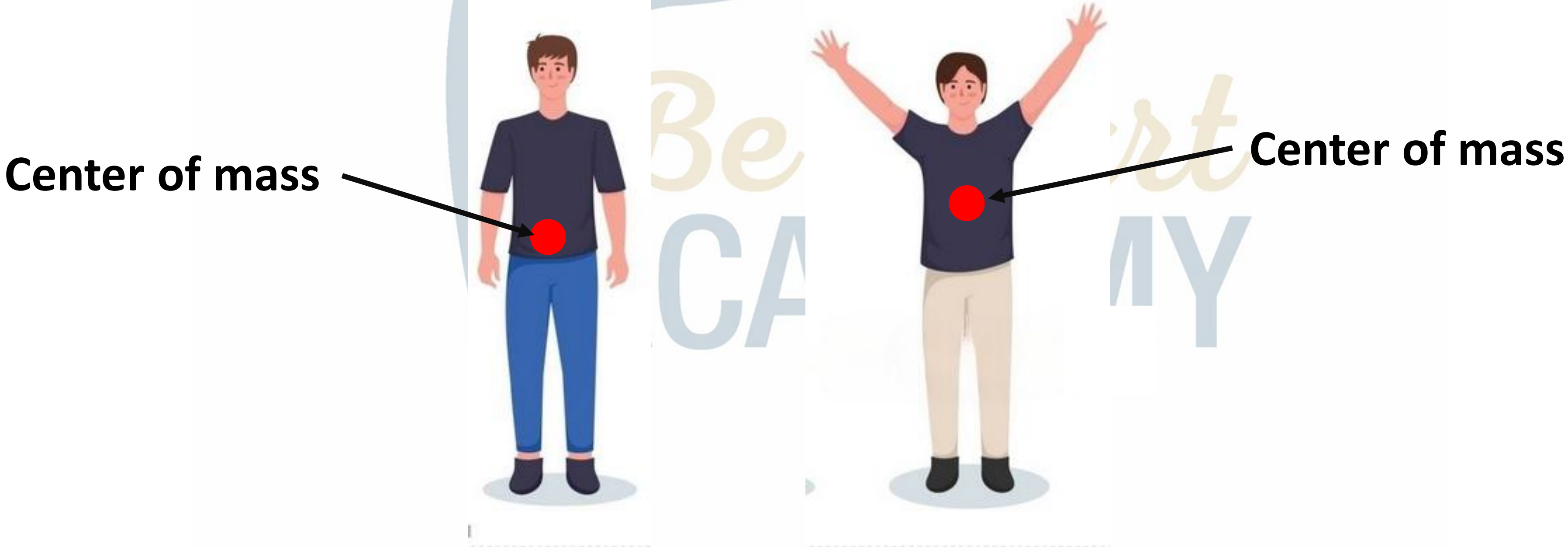
The center of mass of a system (body) is the point at which we can imagine all the mass of a system is to be concentrated.

The center of mass «CM» of a system is a point that represents the average location where the total mass of the system is concentrated

Center of mass «CM»

If an object is considered deformable (their parts can move with respect to each other), the location of the center of mass varies, and depends on the positions of their parts.

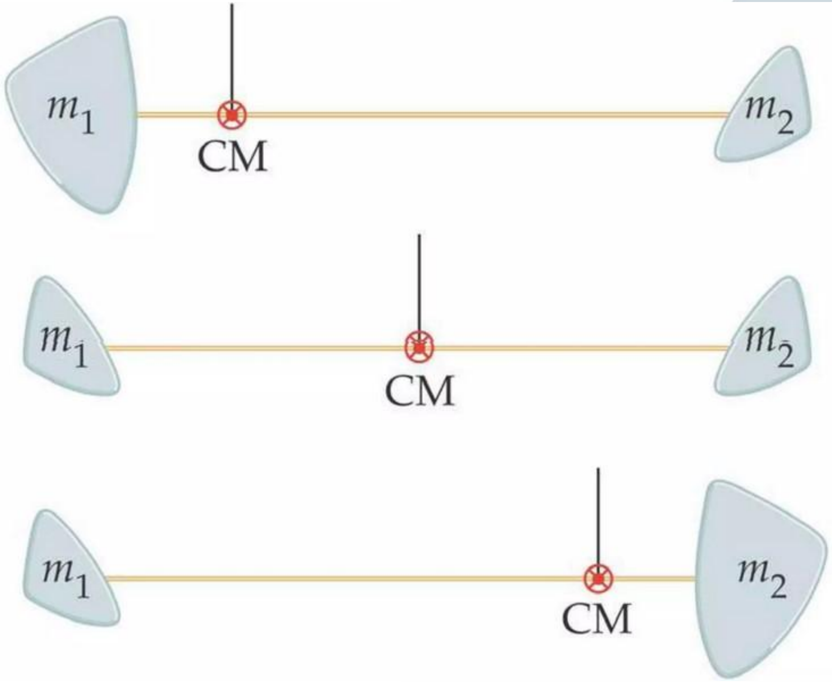
For example, when a man lifts his arms, his center of mass moves to a higher position in his body than when his arms are at his side.



Center of mass «CM» VS Center of gravity «CG»

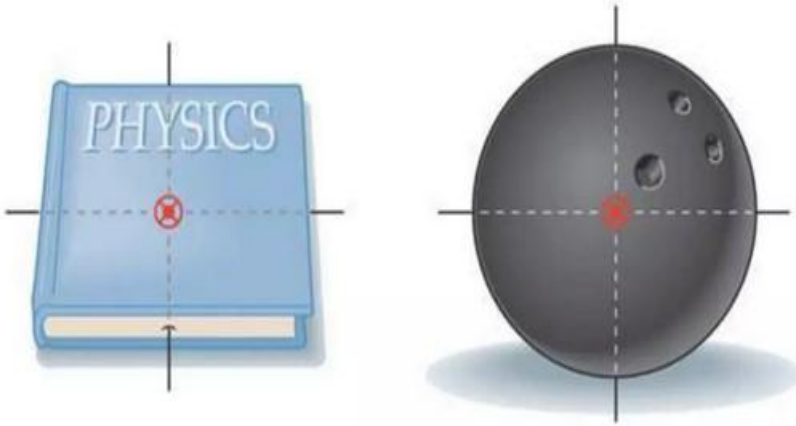
Center of mass «CM»

Means "the average location of all the matter in an object".



Center of gravity «CG»

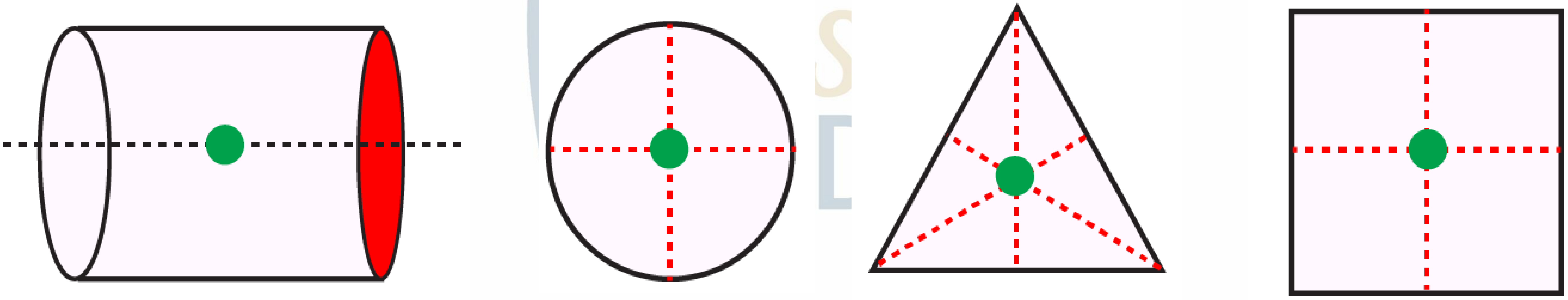
Means "the average location of the force of gravity on an object; it is the point of application of the force of gravity (weight) acting on this system.



Center of mass «CM» VS Center of gravity «CG»

In most practical situations, especially when the gravitational field is uniform, the center of mass and center of gravity can be considered to be at the same point.

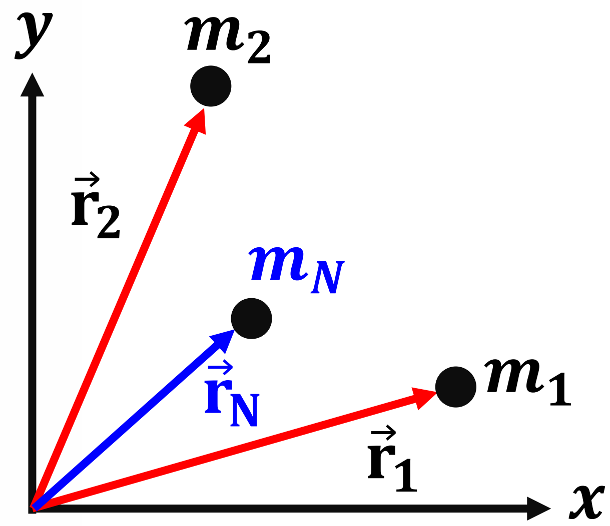
However, they may differ in non-uniform gravitational fields or in cases of large-scale structures like space stations.



Position vector of center of mass

A system consists of N particles of masses $m_1, m_2, m_3, \ldots, m_N$ whose position vectors are $\vec{r}_1, \vec{r}_2, \vec{r}_3, \ldots, \vec{r}_N$ respectively then the position vector of the center of mass is:

$$\vec{r}_G = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_N\vec{r}_N}{m_1 + m_2 + \cdots + m_N}$$



Position vector of center of mass

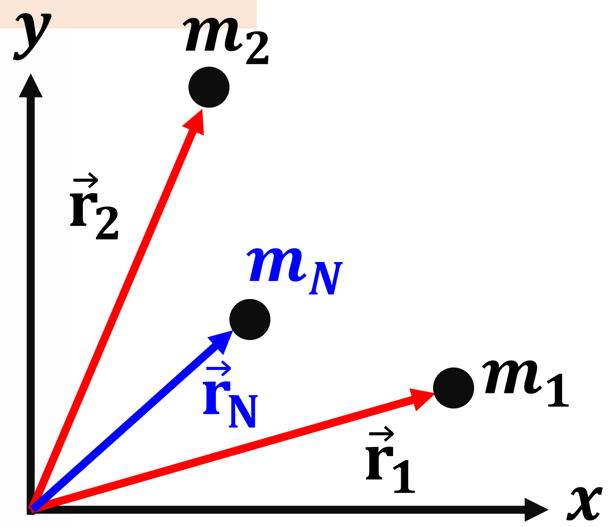
The coordinates of the center of mass are:

$$x_G = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_N x_N}{m_1 + m_2 + \cdots + m_N}$$

$$y_G = \frac{m_1 y_1 + m_2 y_2 + \cdots + m_N y_N}{m_1 + m_2 + \cdots + m_N}$$

The position vector is given by:

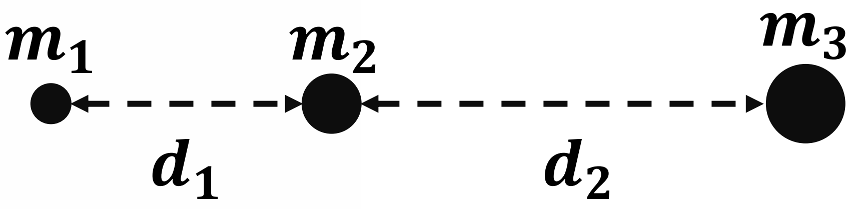
$$\vec{r}_G = x_G.\vec{i} + y_G.\vec{j}$$



Position vector of center of mass

Application 1:

A system consists of three particles of masses $m_1 = 1kg$, $m_2 = 2kg$, and $m_3 = 3kg$ respectively.

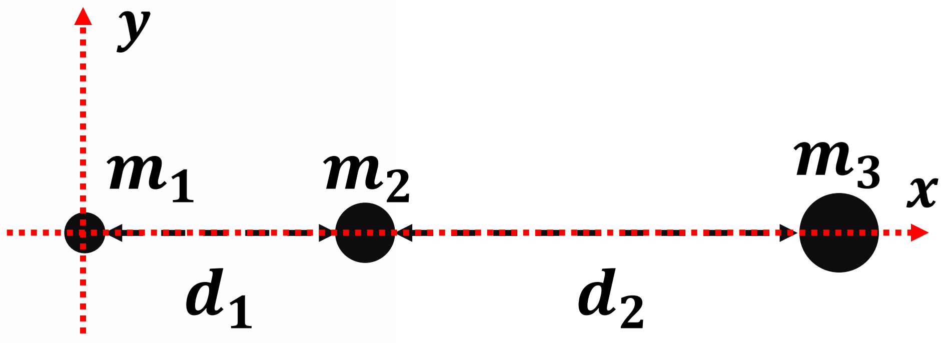


The distances between the particles are $d_1 = 50cm$ and $d_2 = 25cm$ as shown in the figure below.

Determine the position of the center of mass of the system.

Position vector of center of mass

$$x_{COM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$



$$x_{COM} = \frac{1(0) + 2(0.5) + 3(0.75)}{1 + 2 + 3}$$

$$x_{COM} = \frac{0 + 1 + 2.25}{6}$$

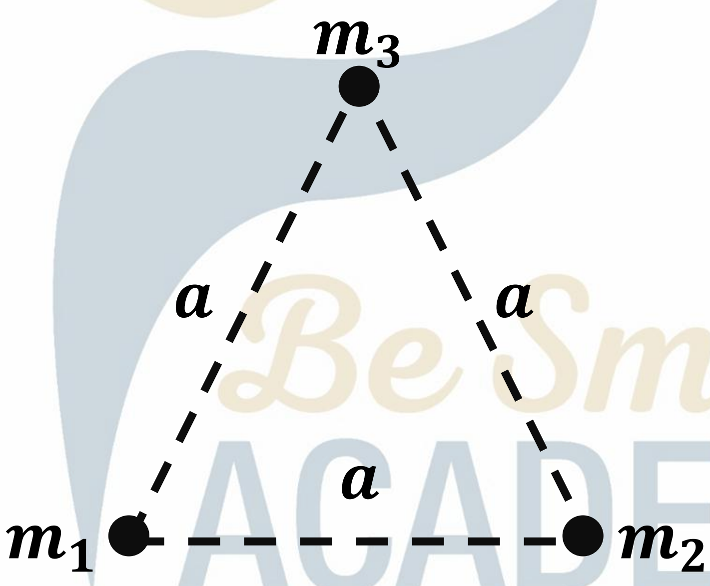
$$x_{COM} = 0.54m$$

$$y_{COM} = 0m$$

Position vector of center of mass

Application 2:

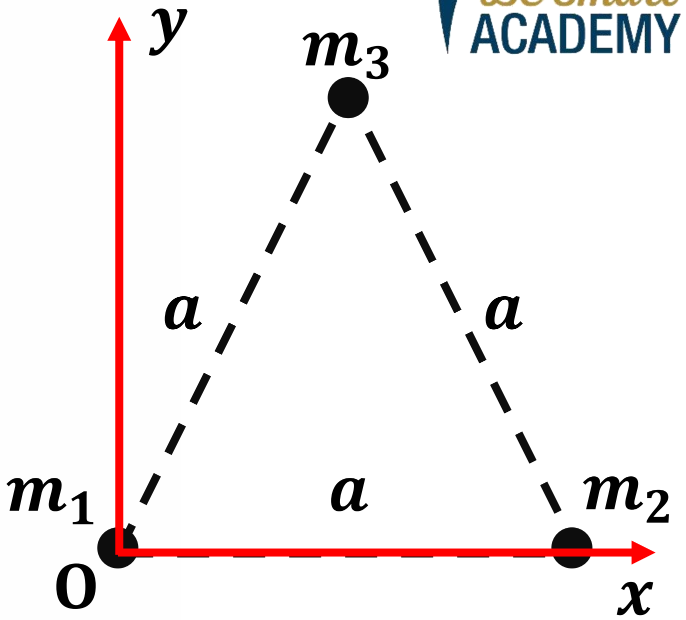
A system consists of three particles of masses $m_1 = m_2 = m_3 = 2kg$ placed at the vertices of a triangle of side $a = 60cm$.



Determine the position of the center of mass of the system

Position vector of center of mass

$$x_{COM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$



$$x_{COM} = \frac{2(0) + 2(0.6) + 2(0.3)}{2 + 2 + 2}$$

$$x_{COM} = \frac{0 + 1.2 + 0.6}{6}$$

$$x_{COM} = 0.3m$$

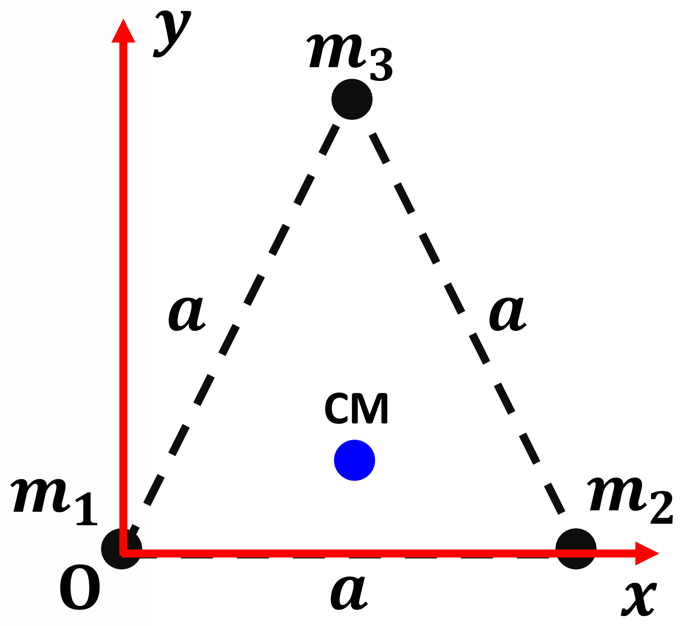
Position vector of center of mass

$$y_{COM} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$y_{COM} = \frac{2(0) + 2(0) + 2(0.5)}{2 + 2 + 2}$$

$$y_{COM} = \frac{0 + 0 + 1}{6}$$

$$\Rightarrow y_{COM} = 0.16m$$



$$\vec{r}_G = x_G.\vec{i} + y_G.\vec{j} \Rightarrow \vec{r}_G = 0.3\vec{i} + 0.16\vec{j}$$

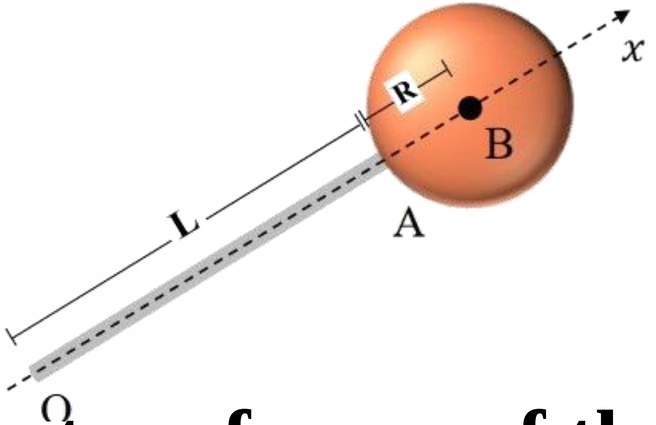
Position vector of center of mass

Application 3:

The aim of this application is to determine the center of mass of a Chupa Chups lollipop!

A lollipop is made of a stick and a solid chunk of candy:

- **The stick is considered to be a uniform thin rod OA of length L = 8 cm and mass $m_1 = 5$g.**
- **The candy is considered to be a uniform sphere of center B with a radius R = 1 cm and mass $m_2 = 25$g.**

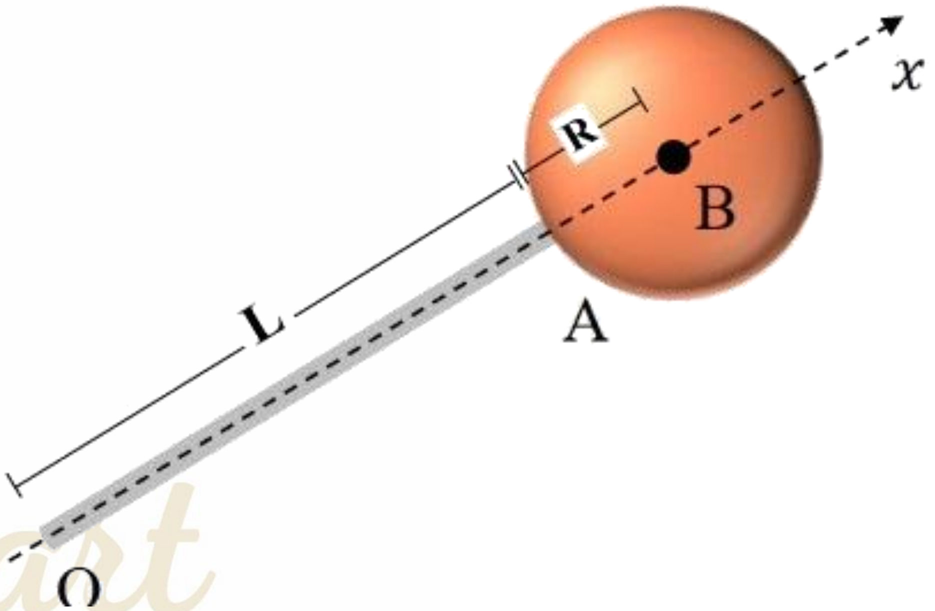


Determine relative to point O the location of the center of mass of the lollipop.

Position vector of center of mass

First, write the coordinates of the center of mass of each object in the system:

$$stick\begin{cases} m_1 = 5g \\ x_1 = \frac{L}{2} = 4cm \\ y_1 = 0 \end{cases}$$



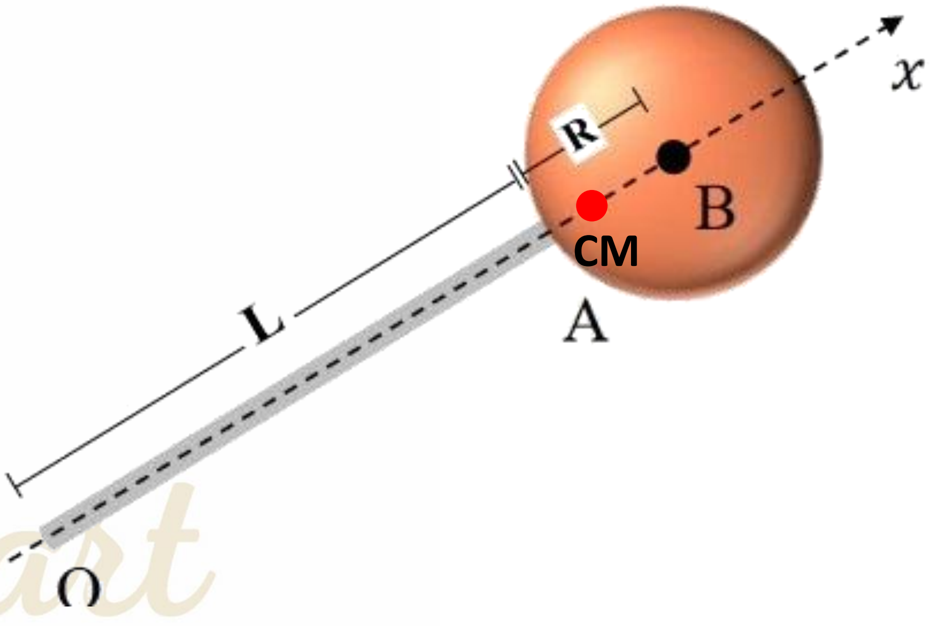
$$Candy\begin{cases} m_2 = 25g \\ x_2 = OB = L + R = 9cm \\ y_2 = 0 \end{cases}$$

Position vector of center of mass

The coordinates of the center of mass of the system are:

$$x_{COM} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

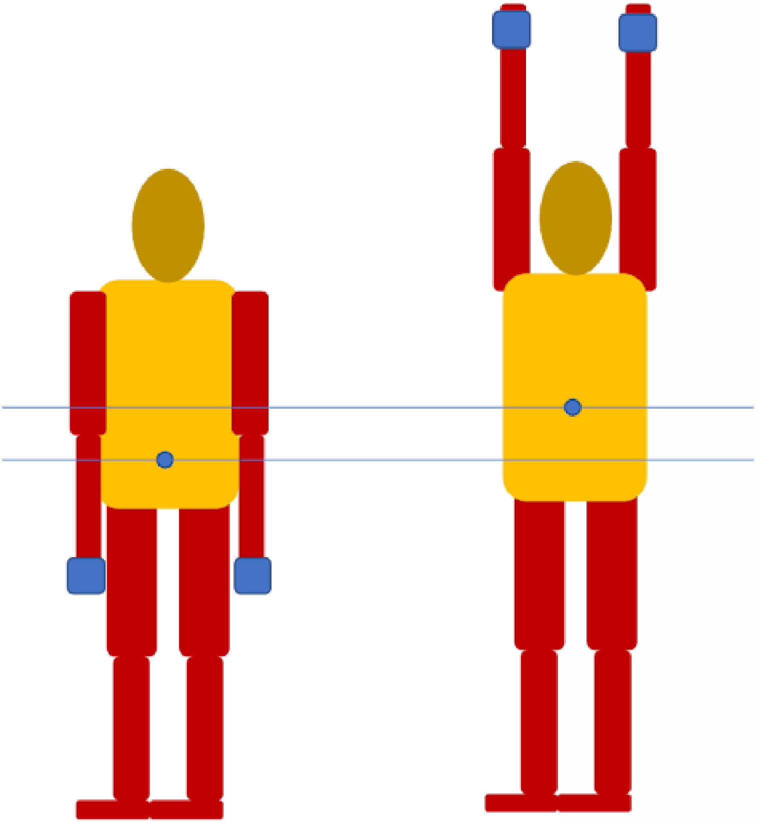
$$x_{COM} = \frac{5(4) + 25(9)}{5 + 25}$$



$$x_{COM} = 8.2cm$$

$$y_{COM} = 0m$$

The End



Grade 11S – Physics

Unit Two: Mechanics

Chapter 9: System of Particles

Prepared & Presented by: Mr. Mohamad Seif

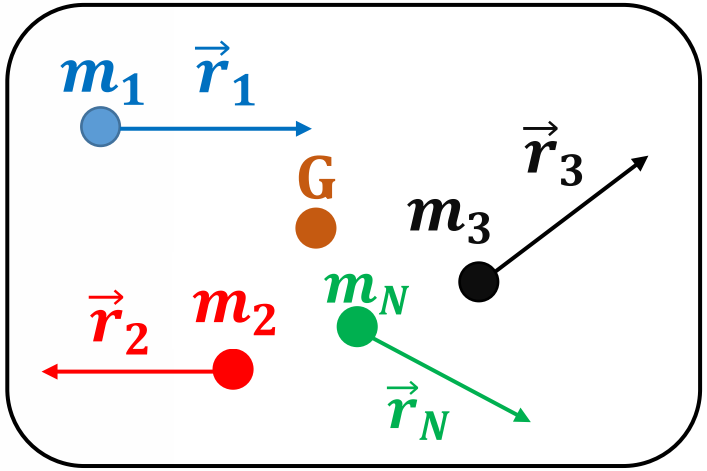
OBJECTIVES

1 **Motion of the center of mass of system of particles**

Motion of the center of mass of system of particles

Consider a system with N parts of constant masses m_1, m_2, $m_3 \dots m_N$ and of position vectors $\vec{r}_1, \vec{r}_2, \vec{r}_3 \dots \vec{r}_N$.

The position vector ($\vec{r}_{CM}$) of the center of mass of a system is:



$$\vec{r}_G = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 \dots + m_N\vec{r}_N}{m_1 + m_2 + \cdots + m_N}$$

Where $M = m_1 + m_2 + \cdots + m_N$

$$M.\vec{r}_G = m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_N\vec{r}_N$$

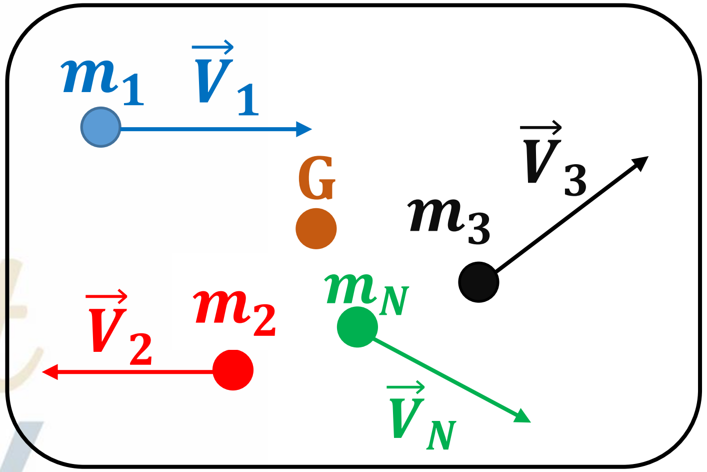
Motion of the center of mass of system of particles

$$M.\vec{r}_G = m_1\vec{r}_1 + m_2\vec{r}_2 + \cdots + m_N\vec{r}_N$$

Derive both sides with respect to time t:

$$M.\vec{v}_G = m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_N\vec{v}_N$$

$$\vec{v}_G = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + \cdots + m_N\vec{v}_N}{M}$$



Theorem of the center of mass of system of particles

$$M.\vec{v}_G = m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 + \cdots + m_N\vec{v}_N$$

Derive both sides with respect to time t:

$$M.\vec{a}_G = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \cdots + m_N\vec{a}_N$$

According to newton's second law $\sum \vec{F}_{ex} = m\vec{a}$ then:

$$\sum \vec{F}_{ex/CM} = \sum \vec{F}_{ex/m_1} + \sum \vec{F}_{ex/m_2} + \sum \vec{F}_{ex/m_3} + \cdots + \sum \vec{F}_{ex/m_N}$$

Motion of the center of mass of system of particles

Application 4:

The position vectors of two particles A and B of masses $m_1 = 50g$ & $m_2 = 100\text{g}$ are respectively $\overrightarrow{\text{OA}} = \vec{\text{r}}_1 = -5\text{t}\vec{\text{i}} + 6\text{t}^2\vec{\text{j}}$ and $\overrightarrow{\text{OB}} = \vec{\text{r}}_2 = (2.5 + 3)\text{t}\vec{\text{i}} + 1.5\text{t}^2\vec{\text{j}}$ in SI units .

1.Show that the position vector of the center of mass G of the system $[m_1, m_2]$, at any instant t, is given by $\overrightarrow{\text{O}G} = \vec{\text{r}}_G = 2\vec{\text{i}} + 3\text{t}^2\vec{\text{j}}$.

2. Determine the resultant external forces acting on the system. Is the system isolated? Why?

Motion of the center of mass of system of particles

$m_1 = 50g$ & $m_2 = 100g$, $\vec{r}_1 = -5t\vec{i} + 6t^2\vec{j}$ & $\vec{r}_2 = (2.5t + 3)\vec{i} + 1.5t^2\vec{j}$

1.Show that the position vector of the center of mass G of the system $[m_1, m_2]$, at any instant t, is given by $\overrightarrow{OG} = \vec{r}_G = 2\vec{i} + 3t^2\vec{j}$.

$$\overrightarrow{OG} = \vec{r}_G = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1 + m_2} = \frac{50\left(-5t\vec{i} + 6t^2\vec{j}\right) + 100\left[(2.5t + 3)\vec{i} + 1.5t^2\vec{j}\right]}{50 + 100}$$

$$\overrightarrow{OG} = \vec{r}_G = \frac{-250t\vec{i} + 300t^2\vec{j} + 250t\vec{i} + 300\vec{i} + 150t^2\vec{j}}{50 + 100}$$

$$\overrightarrow{OG} = \vec{r}_G = \frac{300\vec{i} + 450t^2\vec{j}}{150}$$

$$\overrightarrow{OG} = \vec{r}_G = 2\vec{i} + 3t^2\vec{j}$$

Motion of the center of mass of system of particles

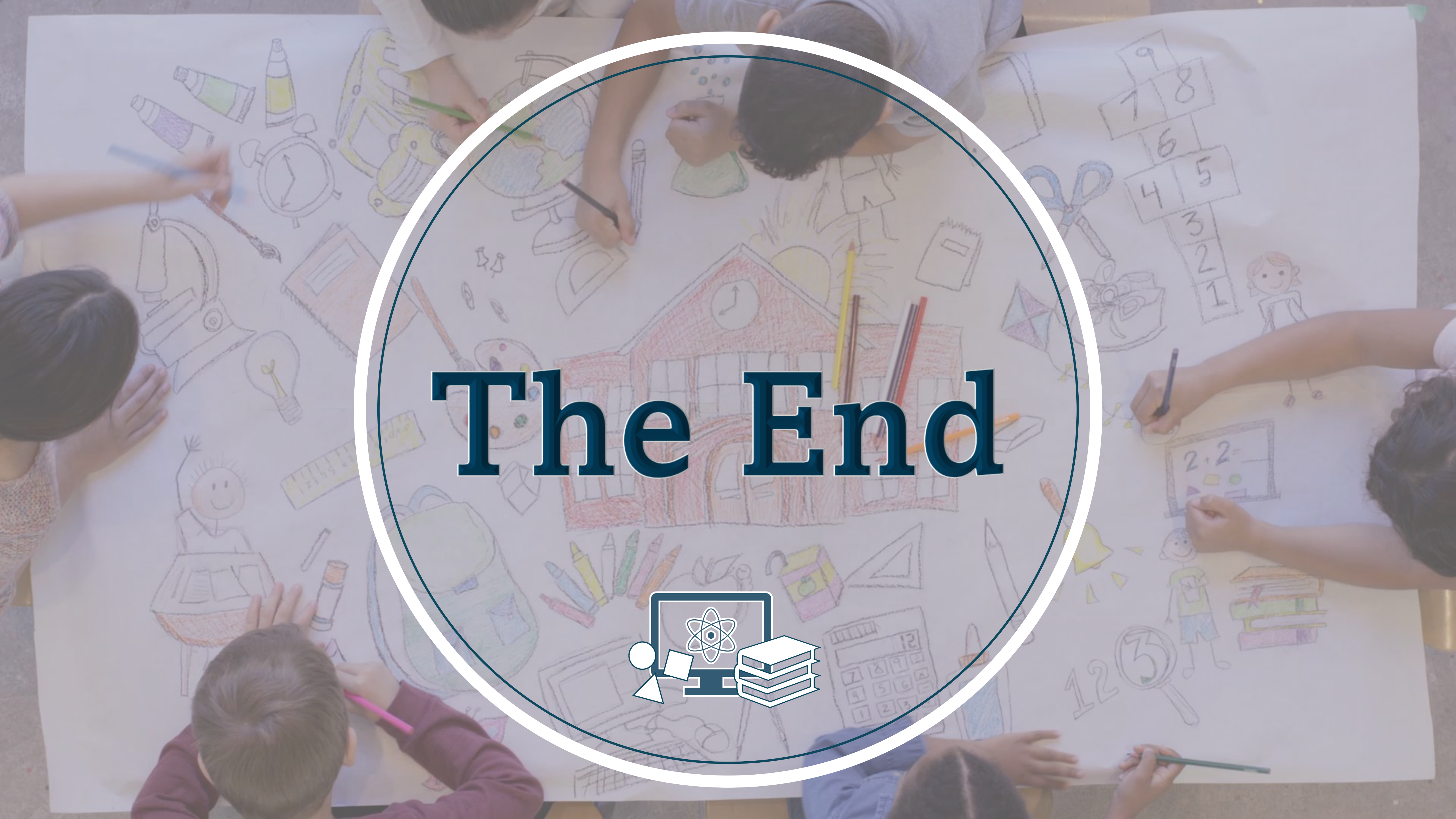
2.Determine the resultant external forces acting on the system. Is the system isolated? Why?

$$\vec{r}_G = 2\vec{i} + 3t^2\vec{j} \Rightarrow \vec{v}_G = 0\vec{i} + 6t\vec{j} \Rightarrow \vec{a}_G = 6\vec{j}$$

$$\sum \vec{F}_{ex} = M.\vec{a}_G \Rightarrow \sum \vec{F}_{ex} = (0.05 + 0.1) \times 6\vec{j}$$

$$\sum \vec{F}_{ex} = 0.9\vec{j} \neq 0 \Rightarrow$$

The system is not isolated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